Intermediate Mathematics (Grade 7 to Grade 10)
Strands & Expectations in the Ontario Curriculum

Note to user: This concept map is designed to assist pre-service and in-service teachers develop a research strategy to locate library materials relevant to their daily lessons. It should not be used in place of the Ontario curriculum. Please visit the Ontario Ministry of Education website – www.edu.gov.on.ca – to access the Ontario curriculum online.

Grade Seven (Ages 12-13)

Number Sense and Numeration

A.1. Representing, comparing, and ordering numbers, including integers
   • Representing, comparing, and ordering decimals to hundredths and fractions, using a variety of tools
   • Generating multiples and factors, using a variety of tools and strategies
   • Identifying and comparing integers found in real-life contexts
   • Representing and ordering integers, using a variety of tools
   • Selecting and justifying the most appropriate representation of a quantity for a given context
   • Representing perfect squares and square roots, using a variety of tools
   • Explaining the relationship between exponential notation and the measurement of area and volume

A.2. Demonstrating an understanding of addition and subtraction of fractions and integers, and applying a variety of computational strategies to solve problems involving whole numbers and decimal numbers
   • Dividing whole numbers by simple fractions and by decimal numbers to hundredths, using concrete materials
   • Using a variety of mental strategies to solve problems involving the addition and subtraction of fractions and decimals
   • Solving problems involving the multiplication and division of decimal numbers to thousandths by one-digit whole numbers, using a variety of tools and strategies
   • Solving multi-step problems arising from real-life contexts and involving whole numbers and decimals, using a variety of tools and strategies
• Using estimation when solving problems involving operations with whole numbers, decimals, and percents, to help judge the reasonableness of a solution
• Evaluating expressions that involve whole numbers and decimals, including expressions that contain brackets, using order of operations
• Adding and subtracting fractions with simple like and unlike denominators, using a variety of tools and algorithms
• Demonstrating, using concrete materials, the relationship between the repeated addition of fractions and the multiplication of that fraction by a whole number
• Adding and subtracting integers, using a variety of tools
• Demonstrating an understanding of proportional relationships using percent, ratio, and rate
• Determining, through investigation, the relationships among fractions, decimals, percents, and ratios
• Solving problems that involve determining whole number percents, using a variety of tools
• Demonstrating an understanding of rate as a comparison, or ratio, of two measurements with different units
• Solving problems involving the calculation of unit rates

Measurement

B.1. Reporting on research into real-life applications of area measurements
• Researching and reporting on real-life applications of area measurements

B.2. Determining the relationships among units and measurable attributes, including the area of a trapezoid and the volume of a right prism
• Sketching different polygonal prisms that share the same volume
• Solving problems that require conversion between metric units of measure
• Solving problems that require conversion between metric units of area
• Determining, through investigation using a variety of tools and strategies, the relationship for calculating the area of a trapezoid, and generalize to develop the formula
• Solving problems involving the estimation and calculation of the area of a trapezoid
• Estimating and calculating the area of composite two-dimensional shapes by decomposing into shapes with known area relationships
• Determining, through investigation using a variety of tools and strategies, the
relationship between the height, the area of the base, and the volume of right prisms with simple polygonal bases, and generalize to develop the formula
• Determining, through investigation using a variety of tools, the surface area of right prisms
• Solving problems that involve the surface area and volume of right prisms and that require conversion between metric measures of capacity and volume

Measurement

B.1. Reporting on research into real-life applications of area measurements
• Researching and reporting on real-life applications of area measurements

B.2. Determining the relationships among units and measurable attributes, including the area of a trapezoid and the volume of a right prism
• Sketching different polygonal prisms that share the same volume
• Solving problems that require conversion between metric units of measure
• Solving problems that require conversion between metric units of area
• Determining, through investigation using a variety of tools and strategies, the relationship for calculating the area of a trapezoid, and generalize to develop the formula
• Solving problems involving the estimation and calculation of the area of a trapezoid
• Estimating and calculating the area of composite two-dimensional shapes by decomposing into shapes with known area relationships
• Determining, through investigation using a variety of tools and strategies, the relationship between the height, the area of the base, and the volume of right prisms with simple polygonal bases, and generalize to develop the formula
• Determining, through investigation using a variety of tools, the surface area of right prisms
• Solving problems that involve the surface area and volume of right prisms and that require conversion between metric measures of capacity and volume

Geometry and Spatial Sense

C.1. Constructing related lines, and classifying triangles, quadrilaterals, and prisms
• Constructing related lines, using angle properties and a variety of tools
• Sorting and classifying triangles and quadrilaterals by geometric properties related to symmetry, angles, and sides, through investigation using a variety of tools and strategies
• Constructing angle bisectors and perpendicular bisectors, using a variety of tools and strategies, and represent equal angles and equal lengths using mathematical notation
• Investigating, using concrete materials, the angles between the faces of a prism, and identify right prisms

C.2. Developing an understanding of similarity, and distinguish similarity and congruence
• Identifying, through investigation, the minimum side and angle information
• Determining, through investigation using a variety of tools
• Demonstrating an understanding that enlarging or reducing two-dimensional shapes creates similar shapes
• Distinguishing between and compare similar shapes and congruent shapes, using a variety of tools and strategies

C.3. Describing location in the four quadrants of a coordinate system; dilatate two-dimensional shapes; and applying transformations to create and analyse designs
• Plotting points using all four quadrants of the Cartesian coordinate plane
• Identifying, performing, and describing dilatations, through investigation using a variety of tools
• Creating and analyzing designs involving translations, reflections, dilatations, and/or simple rotations of two-dimensional shapes, using a variety of tools
• Determining, through investigation using a variety of tools, polygons or combinations of polygons that tile a plane, and describe the transformation(s) involved

Patterning and Algebra

D.1. Representing linear growing patterns (where the terms are whole numbers) using concrete materials, graphs, and algebraic expressions
• Representing linear growing patterns, using a variety of tools, and strategies
• Making predictions about linear growing patterns, through investigation with concrete materials
• Developing and representing the general term of a linear growing pattern, using algebraic expressions involving one operation
• Comparing pattern rules that generate a pattern by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term with pattern rules that use the term number to describe the general term

D.2 Modeling real-life linear relationships graphically and algebraically, and solving simple algebraic equations using a variety of strategies, including inspection and guess and check
• Modeling real-life relationships involving constant rates where the initial condition starts at 0, through investigation using tables of values and graphs
• Modeling real-life relationships involving constant rates, using algebraic equations with variables to represent the changing quantities in the relationship
• Translating phrases describing simple mathematical relationships into algebraic expressions, using concrete materials
• Evaluating algebraic expressions by substituting natural numbers for the variables
• Making connections between evaluating algebraic expressions and determining the term in a pattern using the general term
• Solving linear equations of the form $ax = c$ or $c = ax$ and $ax + b = c$ or variations such as $b + ax = c$ and $c = bx + a$ by modeling with concrete materials, by inspection, or by guess and check, with and without the aid of a calculator

Data Management

E.1. Collecting and organizing categorical, discrete, or continuous primary data and secondary data and displaying the data using charts and graphs, including relative frequency tables and circle graphs
• Collecting data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject and record observations or measurements
• Collecting and organizing categorical, discrete, or continuous primary data and secondary data and display the data in charts, tables, and graphs that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools
• Selecting an appropriate type of graph to represent a set of data, graph the data using technology, and justifying the choice of graph
• Distinguishing between a census and a sample from a population
• Identifying bias in data collection methods
E.2. Making and evaluating convincing arguments, based on the analysis of data

- Reading, interpreting, and drawing conclusions from primary data and from secondary data presented in charts, tables, and graphs
- Identifying, through investigation, graphs that present data in misleading ways
- Determining, through investigation, the effect on a measure of central tendency of adding or removing a value or values
- Identifying and describing trends, based on the distribution of the data presented in tables and graphs, using informal language
- Making inferences and convincing arguments that are based on the analysis of charts, tables, and graphs

E.3. Comparing experimental probabilities with the theoretical probability of an outcome involving two independent events

- Researching and reporting on real-world applications of probabilities expressed in fraction, decimal, and percent form
- Making predictions about a population when given a probability
- Representing in a variety of ways all the possible outcomes of a probability experiment involving two independent events, and determine the theoretical probability of a specific outcome involving two independent events
- Performing a simple probability experiment involving two independent events, and comparing the experimental probability with the theoretical probability of a specific outcome
Grade Eight (Ages 13-14)

Number Sense and Numeration

A.1. Representing, comparing, and ordering equivalent representations of numbers, including those involving positive exponents
• Expressing repeated multiplication using exponential notation
• Representing whole numbers in expanded form using powers of ten
• Representing, comparing, and ordering rational numbers
• Translating between equivalent forms of a number
• Determining common factors and common multiples using the prime factorization of numbers

A.2. Solving problems involving whole numbers, decimal numbers, fractions, and integers, using a variety of computational strategies
• Solving multi-step problems arising from real-life contexts and involving whole numbers and decimals, using a variety of tools and strategies
• Solving problems involving percents expressed to one decimal place and whole-number percents greater than 100
• Using estimation when solving problems involving operations with whole numbers, decimals, percents, integers, and fractions, to help judge the reasonableness of a solution
• Representing the multiplication and division of fractions, using a variety of tools and strategies
• Evaluating expressions that involve integers, including expressions that contain brackets and exponents, using order of operations
• Multiplying and dividing decimal numbers by various powers of ten
• Estimating, and verifying using a calculator, the positive square roots of whole numbers, and distinguishing between whole numbers that have whole-number square roots and those that do not

A.3. Solving problems by using proportional reasoning in a variety of meaningful contexts
• Identifying and describing real-life situations involving two quantities that are directly proportional
• Solving problems involving proportions, using concrete materials, drawings, and
variables
- Solving problems involving percent that arise from real-life contexts
- Solving problems involving rates

Measurements

B.1. Researching, describing, and reporting on applications of volume and capacity measurement
- Researching, describing, and reporting on applications of volume and capacity measurement

B.2. Determining the relationships among units and measurable attributes, including the area of a circle and the volume of a cylinder
- Solving problems that require conversions involving metric units of area, volume, and capacity
- Measuring the circumference, radius, and diameter of circular objects, using concrete materials
- Determining, through investigation using a variety of tools and strategies, the relationships for calculating the circumference and the area of a circle, and generalize to develop the formulas
- Solving problems involving the estimation and calculation of the circumference and the area of a circle
- Determining, through investigation using a variety of tools and strategies
- Determining, through investigation using concrete materials, the surface area of a cylinder
- Solving problems involving the surface area and the volume of cylinders, using a variety of strategies

Geometry and Spatial Sense

C.1. Demonstrating an understanding of the geometric properties of quadrilaterals and circles and the applications of geometric properties in the real world
- Sorting and classifying quadrilaterals by geometric properties, including those based on diagonals, through investigation using a variety of tools
- Constructing a circle, given its centre and radius, or its centre and a point on the circle, or three points on the circle
- Investigating and describing applications of geometric properties in the real world
C.2. Developing geometric relationships involving lines, triangles, and polyhedra, and solving problems involving lines and triangles
• Determining, through investigation using a variety of tools, relationships among area, perimeter, corresponding side lengths, and corresponding angles of similar shapes
• Determining, through investigation using a variety of tools, the angle relationships for intersecting lines and for parallel lines and transversals, and the sum of the angles of a triangle
• Solving angle-relationship problems involving triangles, intersecting lines, and parallel lines and transversals
• Determining the Pythagorean relationship, through investigation using a variety of tools and strategies
• Solving problems involving right triangles geometrically, using the Pythagorean relationship
• Determining, through investigation using concrete materials, the relationship between the numbers of faces, edges, and vertices of a polyhedron

C.3. Representing transformations using the Cartesian coordinate plane, and making connections between transformations and the real world
• Graphing the image of a point, or set of points, on the Cartesian coordinate plane after applying a transformation to the original point(s)
• Identifying, through investigation, real-world movements that are translations, reflections, and rotations

Patterning and Algebra

D.1. Representing linear growing patterns using graphs, algebraic expressions, and equations
• Representing, through investigation with concrete materials, the general term of a linear pattern, using one or more algebraic expressions
• Representing linear patterns graphically, using a variety of tools
• Determining a term, given its term number, in a linear pattern that is represented by a graph or an algebraic equation

D.2. Modeling linear relationships graphically and algebraically, and solving and verifying algebraic equations, using a variety of strategies, including inspection, guess and check, and using a “balance” model
• Describing different ways in which algebra can be used in real-life situations
• Modeling linear relationships using tables of values, graphs, and equations, through investigation using a variety of tools
• Translating statements describing mathematical relationships into algebraic expressions and equations
• Evaluating algebraic expressions with up to three terms, by substituting fractions, decimals, and integers for the variables
• Making connections between solving equations and determining the term number in a pattern, using the general term
• Solving and verifying linear equations involving a one-variable term and having solutions that are integers, by using inspection, guess and check, and a “balance” model

Data Management

E.1. Collecting and organizing categorical, discrete, or continuous primary data and secondary data and displaying the data using charts and graphs, including frequency tables with intervals, histograms, and scatter plots
• Collecting data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements
• Organizing into intervals a set of data that is spread over a broad range
• Collecting and organizing categorical, discrete, or continuous primary data and secondary data, and displaying the data in charts, tables, and graphs that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools
• Selecting an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph
• Explaining the relationship between a census, a representative sample, sample size, and a population

E.2. Applying a variety of data management tools and strategies to make convincing arguments about data
• Reading, interpreting, and drawing conclusions from primary data and from secondary data, presented in charts, tables, and graphs
• Determining, through investigation, the appropriate measure of central tendency needed to compare sets of data
• Demonstrating an understanding of the appropriate uses of bar graphs and histograms by comparing their characteristics
• Comparing two attributes or characteristics, using a scatter plot, and determining whether or not the scatter plot suggests a relationship
• Identifying and describing trends, based on the rate of change of data from tables and graphs, using informal language
• Making inferences and convincing arguments that are based on the analysis of charts, tables, and graphs
• Comparing two attributes or characteristics, using a variety of data management tools and strategies

E.3. Using probability models to make predictions about real-life events
• Comparing, through investigation, the theoretical probability of an event with experimental probability, and explain why they might differ
• Determining, through investigation, the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases, using class-generated data and technology-based simulation models
• Identifying the complementary event for a given event, and calculate the theoretical probability that a given event will not occur
Number Sense and Algebra

A.1. Demonstrating an understanding of the exponent rules of multiplication and division, and applying them to simplify expressions
  • Substituting into and evaluating algebraic expressions involving exponents
  • Describing the relationship between the algebraic and geometric representations of a single-variable term up to degree three
  • Deriving, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents
  • Extending the multiplication rule to derive and understand the power of a power rule, and applying it to simplify expressions involving one and two variables with positive exponents

A.2. Manipulating numerical and polynomial expressions, and solving first-degree equations.
  • Simplifying numerical expressions involving integers and rational numbers, with and without the use of technology
  • Solving problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion
  • Relating their understanding of inverse operations to squaring and taking the square root, and applying inverse operations to simplify expressions and solve equations
  • Adding and subtracting polynomials with up to two variables, using a variety of tools
  • Multiplying a polynomial by a monomial involving the same variable, using a variety of tools
  • Expanding and simplifying polynomial expressions involving one variable, using a variety of tools
  • Solving first-degree equations, including equations with fractional coefficients, using a variety of tools and strategies
  • Rearranging formulas involving variables in the first degree, with and without substitution
• Solving problems that can be modeled with first-degree equations, and compare algebraic methods to other solution methods

**Linear Relations**

**B.1. Applying data-management techniques to investigate relationships between two variables**
- Interpreting the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant
- Posing problems, identifying variables, and formulating hypotheses associated with relationships between two variables
- Designing and carrying out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology
- Describing trends and relationships observed in data, making inferences from data, comparing the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses

**B.2. Demonstrating an understanding of the characteristics of a linear relation**
- Constructing tables of values, graphs, and equations, using a variety of tools, to represent linear relations derived from descriptions of realistic situations
- Constructing tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools, for linearly related and non-linearly related data collected from a variety of sources
- Identifying, through investigation, some properties of linear relations, and applying these properties to determine whether a relation is linear or non-linear
- Comparing the properties of direct variation and partial variation in applications, and identifying the initial value
- Determining the equation of a line of best fit for a scatter plot, using an informal process

**B.3. Connecting various representations of a linear relation**
- Determining values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation
- Describing a situation that would explain the events illustrated by a given graph of a relationship between two variables
- Determining other representations of a linear relation, given one representation
• Describing the effects on a linear graph and make the corresponding changes to
the linear equation when the conditions of the situation they represent are varied

Analytic Geometry

C.1. Determining the relationship between the form of an equation and the
shaping of its graph with respect to linearity and non-linearity
• Determining, through investigation, the characteristics that distinguish the
equation of a straight line from the equations of nonlinear relations
• Identifying, through investigation, the equation of a line in any of the forms \( y = mx + b, Ax + By + C = 0, x = a, y = b \)
• Expressing the equation of a line in the form \( y = mx + b \), given the form \( Ax + By + C = 0 \)

C.2. Determining, through investigation, the properties of the slope and y-
intercept of a linear relation
• Determining, through investigation, various formulas for the slope of a line
segment or to determine the slope of a line segment or a line
• Identifying, through investigation with technology, the geometric significance of
\( m \) and \( b \) in the equation \( y = mx + b \);
• Determining, through investigation, connections among the representations of a
constant rate of change of a linear relation
• Identifying, through investigation, properties of the slopes of lines and line
segments, using graphing technology to facilitate investigations, where
appropriate.

C.3. Solving problems involving linear relations
• Graphing lines by hand, using a variety of techniques
• Determining the equation of a line from information about the line
• Describing the meaning of the slope and y-intercept for a linear relation arising
from a realistic situation, and describe a situation that could be modelled by a
given linear equation
• Identifying and explaining any restrictions on the variables in a linear relation
arising from a realistic situation
• Determining graphically the point of intersection of two linear relations, and
interpret the intersection point in the context of an application

Measurement and Geometry
D.1. Determining, through investigation, the optimal values of various measurements
• Determining the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools, and by examining various values of the area as the side lengths change and the perimeter remains constant
• Determining the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools, and by examining various values of the side lengths and the perimeter as the area stays constant
• Identifying, through investigation with a variety of tools, the effect of varying the dimensions on the surface area of square-based prisms and cylinders, given a fixed volume
• Explaining the significance of optimal area, surface area, or volume in various applications
• Posing and solving problems involving maximization and minimization of measurements of geometric shapes and figures

D.2. Solving problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures
• Relating the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$
• Solving problems using the Pythagorean theorem, as required in applications
• Solving problems involving the areas and perimeters of composite two-dimensional shapes
• Developing, through investigation, the formulas for the volume of a pyramid, a cone, and a sphere
• Determining, through investigation, the relationship for calculating the surface area of a pyramid
• Solving problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures

D.3. Verifying, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems
• Illustrating equivalent ratios, using a variety of tools
• Representing, using equivalent ratios and proportions, directly proportional relationships arising from realistic situations
• Solving for the unknown value in a proportion, using a variety of methods
• Making comparisons using unit rates
• Solving problems involving ratios, rates, and directly proportional relationships in various contexts, using a variety of methods
• Solving problems requiring the expression of percents, fractions, and decimals in their equivalent forms
Grade Nine (Ages 14-15)
Applied Stream

Number Sense and Algebra

A.1. Solving problems involving proportional reasoning
• Illustrating equivalent ratios, using a variety of tools
• Representing, using equivalent ratios and proportions, directly proportional
  relationships arising from realistic situations
• Solving for the unknown value in a proportion, using a variety of methods
• Making comparisons using unit rates
• Solving problems involving ratios, rates, and directly proportional relationships
  in various contexts, using a variety of methods
• Solving problems requiring the expression of percents, fractions, and decimals in
  their equivalent forms

A.2. Simplifying numerical and polynomial expressions in one variable, and
solving simple first-degree equations
• Simplifying numerical expressions involving integers and rational numbers, with
  and without the use of technology
• Relating their understanding of inverse operations to squaring and taking the
  square root, and applying inverse operations to simplify expressions and solve
  equations
• Describing the relationship between the algebraic and geometric representations
  of a single-variable term up to degree three
• Substituting into and evaluating algebraic expressions involving exponents
• Adding and subtracting polynomials involving the same variable up to degree
  three, using a variety of tools
• Multiplying a polynomial by a monomial involving the same variable to give
  results up to degree three, using a variety of tools
• Solving first-degree equations with nonfractional coefficients, using a variety of
  tools and strategies
• Substituting into algebraic equations and solve for one variable in the first degree

Linear Relations
B.1. Applying data-management techniques to investigate relationships between two variables
• Interpreting the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant
• Posing problems, identifying variables, and formulating hypotheses associated with relationships between two variables
• Carrying out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology
• Describing trends and relationships observed in data, making inferences from data, comparing the inferences with hypotheses about the data, and explaining any differences between the inferences and the hypotheses

B.2. Determining the characteristics of linear relations
• Constructing tables of values and graphs, using a variety of tools, to represent linear relations derived from descriptions of realistic situations
• Constructing tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools, for linearly related and non-linearly related data collected from a variety of sources
• Identifying, through investigation, some properties of linear relations

B.3. Demonstrating an understanding of constant rate of change and its connection to linear relations
• Determining, through investigation, that the rate of change of a linear relation can be found by choosing any two points on the line that represents the relation, finding the vertical change between the points and the horizontal change between the points, and writing the ratio
• Determining, through investigation, connections among the representations of a constant rate of change of a linear relation
• Comparing the properties of direct variation and partial variation in applications, and identifying the initial value
• Expressing a linear relation as an equation in two variables, using the rate of change and the initial value
• Describing the meaning of the rate of change and the initial value for a linear relation arising from a realistic situation

B.4. Connecting various representations of a linear relation, and solve problems using the representations
• Determining values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation
• Describing a situation that would explain the events illustrated by a given graph of a relationship between two variables
• Determining other representations of a linear relation arising from a realistic situation, given one representation
• Solving problems that can be modeled with first-degree equations, and comparing the algebraic method to other solution methods
• Describing the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied
• Determining graphically the point of intersection of two linear relations, and interpreting the intersection point in the context of an application
• Selecting a topic involving a two-variable relationship, pose a question on the topic, collecting data to answer the question, and presenting its solution using appropriate representations of the data

Measurement and Geometry

C.1. Determining, through investigation, the optimal values of various measurements of rectangles
• Determining the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools, and by examining various values of the area as the side lengths change and the perimeter remains constant
• Determining the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools, and by examining various values of the side lengths and the perimeter as the area stays constant
• Solving problems that require maximizing the area of a rectangle for a fixed perimeter or minimizing the perimeter of a rectangle for a fixed area

C.2. Solving problems involving the measurements of two-dimensional shapes and the volumes of three-dimensional figures
• Relating the geometric representation of the Pythagorean theorem to the algebraic representation \( a^2 + b^2 = c^2 \)
• Solving problems using the Pythagorean theorem, as required in applications
• Solving problems involving the areas and perimeters of composite two-dimensional shapes
• Developing, through investigation, the formulas for the volume of a pyramid, a cone, and a sphere
• Solving problems involving the volumes of prisms, pyramids, cylinders, cones, and spheres

C.3. Determining, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems
• Determining, through investigation using a variety of tools, and describing the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and applying the results to problems involving the angles of polygons
• Determining, through investigation using a variety of tools, and describing the properties and relationships of the angles formed by parallel lines cut by a transversal, and applying the results to problems involving parallel lines
• Creating an original dynamic sketch, paper folding design, or other illustration that incorporates some of the geometric properties from this section, or finding and reporting on some real-life application(s) of the geometric properties
Grade Ten (Ages 14-15)
Academic Stream

Quadratic Relations

A.1. Determining the basic properties of quadratic relations
• Collecting data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology, or from secondary sources; graph the data and draw a curve of best fit, if appropriate, with or without the use of technology
• Determining, through investigation with and without the use of technology, that a quadratic relation of the form \( y = ax^2 + bx + c \) (\( a \neq 0 \)) can be graphically represented as a parabola, and that the table of values yields a constant second difference
• Identifying the key features of a graph of a parabola, and using the appropriate terminology to describe them
• Comparing, through investigation using technology, the features of the graph of \( y = x^2 \) and the graph of \( y = 2x \), and determining the meaning of a negative exponent and of zero as an exponent

A.2. Relating transformations of the graph of \( y = x^2 \) to the algebraic representation \( y = a(x - h)^2 + k \)
• Identifying, through investigation using technology, the effect on the graph of \( y = x^2 \) of transformations by considering separately each parameter \( a \), \( h \), and \( k \)
• Explaining the roles of \( a \), \( h \), and \( k \) in \( y = a(x - h)^2 + k \), using the appropriate terminology to describe the transformations, and identifying the vertex and the equation of the axis of symmetry
• Sketching, by hand, the graph of \( y = a(x - h)^2 + k \) by applying transformations to the graph of \( y = x^2 \)
• Determining the equation, in the form \( y = a(x - h)^2 + k \), of a given graph of a parabola

A.3. Solving quadratic equations and interpret the solutions with respect to the corresponding relations
• Expanding and simplifying second-degree polynomial expressions, using a variety of tools and strategies
• Factoring polynomial expressions involving common factors, trinomials, and
• Differences of squares, using a variety of tools and strategies
  • Determining, through investigation, and describe the connection between the factors of a quadratic expression and the x-intercepts of the graph of the corresponding quadratic relation, expressed in the form $y = a(x - r)(x - s)$
  • Interpreting real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the x-intercepts of the corresponding relations
  • Expressing $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools
  • Sketching or graphing a quadratic relation whose equation is given in the form $12y = ax^2 + bx + c$, using a variety of methods
  • Exploring the algebraic development of the quadratic formula
  • Solving quadratic equations that have real roots, using a variety of methods

A.4. Solving problems involving quadratic relations
  • Determining the zeros and the maximum or minimum value of a quadratic relation from its graph or from its defining equation
  • Solving problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology

Analytic Geometry

B.1. Modeling and solving problems involving the intersection of two straight lines
  • Solving systems of two linear equations involving two variables, using the algebraic method of substitution or elimination
  • Solving problems that arise from realistic situations described in words or represented by linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method

B.2. Solving problems using analytic geometry involving properties of lines and line segments
  • Developing the formula for the midpoint of a line segment, and using this formula to solve problems
  • Developing the formula for the length of a line segment, and using this formula to solve problems
  • Developing the equation for a circle with centre $(0, 0)$ and radius $r$, by applying the formula for the length of a line segment
• Determining the radius of a circle with centre (0, 0), given its equation; write the equation of a circle with centre (0, 0), given the radius; and sketch the circle, given the equation in the form \(x^2 + y^2 = r^2\)
• Solving problems involving the slope, length, and midpoint of a line segment

B.3. Verifying geometric properties of triangles and quadrilaterals, using analytic geometry
• Determining, through investigation, some characteristics and properties of geometric figures
• Verifying, using algebraic techniques and analytic geometry, some characteristics of geometric figures
• Planning and implementing a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property

Trigonometry

C.1. Using their knowledge of ratio and proportion to investigate similar triangles and solving problems related to similarity
• Verifying, through investigation, the properties of similar triangles
• Describing and comparing the concepts of similarity and congruence
• Solving problems involving similar triangles in realistic situations

C.2. Solving problems involving right triangles, using the primary trigonometric ratios and the Pythagorean theorem
• Determining, through investigation, the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios; hypotenuse
• Determining the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem
• Solving problems involving the measures of sides and angles in right triangles in real life applications, using the primary trigonometric ratios and the Pythagorean theorem

C.3. Solving problems involving acute triangles, using the sine law and the cosine law
• Exploring the development of the sine law within acute
• Exploring the development of the cosine law within acute triangles
• Determining the measures of sides and angles in acute triangles, using the sine
law and the cosine law
• Solving problems involving the measures of sides and angles in acute triangles
Grade Ten (Ages 14-15)
Applied Stream

Measurement and Trigonometry

A.1. Using their knowledge of ratio and proportion to investigate similar triangles and solving problems related to similarity
• Verifying, through investigation, properties of similar triangles
• Determining the lengths of sides of similar triangles, using proportional reasoning
• Solving problems involving similar triangles in realistic situations

A.2. Solving problems involving right triangles, using the primary trigonometric ratios and the Pythagorean theorem
• Determining, through investigation, the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios
• Determining the measures of the sides and angles in right triangles, using the primary opposite hypotenuse trigonometric ratios and the Pythagorean theorem
• Solving problems involving the measures of sides and angles in right triangles in real life applications, using the primary trigonometric ratios and the Pythagorean theorem
• Describing, through participation in an activity, the application of trigonometry in an occupation

A.3. Solving problems involving the surface areas and volumes of three-dimensional figures, and using the imperial and metric systems of measurement
• Using the imperial system when solving measurement problems
• Performing everyday conversions between the imperial system and the metric system and within these systems, as necessary to solve problems involving measurement
• Determining, through investigation, the relationship for calculating the surface area of a pyramid
• Solving problems involving the surface areas of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving combinations of these figures, using the metric system or the imperial system, as appropriate
Modeling Linear Relations

B.1. Manipulating and solving algebraic equations, as needed to solve problems
- Solving first-degree equations involving one variable, including equations with fractional coefficients
- Determining the value of a variable in the first degree, using a formula
- Expressing the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$

B.2. Graphing a line and write the equation of a line from given information
- Connecting the rate of change of a linear relation to the slope of the line, and defining the slope as the ratio $m = \frac{\text{rise}}{\text{run}}$
- Identifying, through investigation, $y = mx + b$ as a common form for the equation of a straight line, and identify the special cases $x = a$, $y = b$; rise run
- Identifying, through investigation with technology, the geometric significance of $m$ and $b$ in the equation $y = mx + b$
- Identifying, through investigation, properties of the slopes of lines and line segments, using graphing technology to facilitate investigations, where appropriate
- Graphing lines by hand, using a variety of techniques
- Determining the equation of a line, given its graph, the slope and $y$-intercept, the slope and a point on the line, or two points on the line

B.3. Solving systems of two linear equations, and solve related problems that arise from realistic situations
- Determining graphically the point of intersection of two linear relations
- Solving systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination
- Solving problems that arise from realistic situations described in words or represented by given linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method

Quadratic Relations

C.1. Manipulating algebraic expressions, as needed to understand quadratic relations
- Expanding and simplifying second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using a variety of tools and strategies
• Factoring binomials and trinomials involving one variable up to degree two, by determining a common factor using a variety of tools and strategies
• Factoring simple trinomials of the form \( x^2 + bx + c \), using a variety of tools and strategies
• Factoring the difference of squares of the form \( x^2 - a^2 \) (e.g., \( x^2 - 16 \))

C.2. Identifying characteristics of quadratic relations
• Collecting data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology, or from secondary sources; graphing the data and draw a curve of best fit, if appropriate, with or without the use of technology
• Determining, through investigation using technology, that a quadratic relation of the form \( y = ax^2 + bx + c \) \((a \neq 0)\) can be graphically represented as a parabola, and determine that the table of values yields a constant second difference
• Identifying the key features of a graph of a parabola, using a given graph or a graph generated with technology from its equation, and use the appropriate terminology to describe the features
• Comparing, through investigation using technology, the graphical representations of a quadratic relation in the form \( y = x^2 + bx + c \) and the same relation in the factored form \( y = (x - r)(x - s) \), and describing the connections between each algebraic representation and the graph

C.3. Solving problems by interpreting graphs of quadratic relations
• Solving problems involving a quadratic relation by interpreting a given graph or a graph generated with technology from its equation
• Solving problems by interpreting the significance of the key features of graphs obtained by collecting experimental data involving quadratic relations